SUPPRESSION OF $0\nu 2\beta$ DECAY FROM CP VIOLATION

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The observed phenomenon of neutrino oscillations is interpreted as the proof that neutrinos must have mass. As this is true for the neutrinos in the mass basis, the mass matrix in the flavor (weak) basis may still contain zeros. This can happen if the CP violating phases, usually neglected, come into play and result in suppression of processes which half-life depends on the masses of ν_e, ν_μ, or ν_τ. In the present paper we investigate the possibility of such suppression of the neutrinoless double beta decay ($0\nu 2\beta$).

1. Introduction

The oscillations of neutrinos have been confirmed by a number of experiments.[1] The usual interpretation of this fact within standard quantum mechanics implies that the quantities $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$ must be non-zero, thus the neutrinos (with the possible exception of the lightest one) must be massive. This calls for need of extending the standard model of particles and interactions (SM), in which neutrinos are assumed to be massless.

Neutrinos reveal also one more characteristic feature, namely, the particles that take part in weak interactions, like the β decay, are not the same particles that propagate through space. In another words, the ν_e, ν_μ, and ν_τ neutrinos, which are produced in weak processes are said to be in the interaction (or flavor) basis, while neutrinos that propagate (eg. from the Sun to Earth, from radioactive source to the detector etc.) are in the physical basis and are labeled by ν_1, ν_2, and ν_3. The states ν_i, $i = 1, 2, 3$, are mass eigenstates, so that one can speak about their masses m_i. The weak eigenstates ν_α, $\alpha = e, \mu, \tau$, are linear combinations of ν_i,

$$\nu_\alpha = U_{\alpha i}\nu_i, \tag{1}$$

and do not possess definite masses. The neutrino phenomenological mass matrix in the flavor basis, $\mathcal{M}$, is usually written as

$$\mathcal{M} = U^* \mathrm{diag}(m_1, m_2, m_3) U^\dagger, \tag{2}$$

where U in Eqs. (1) and (2) is the unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix. For massive Majorana neutrinos it can be conve-

niently parameterized by three mixing angles θ_{12}, θ_{13}, and θ_{23}, two Majorana CP violating phases α_{12}, and α_{13}, and one Dirac CP violating phase δ:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix}$$
$$\times \, \mathrm{diag}(1, e^{\mathrm{i}\alpha_{12}}, e^{\mathrm{i}\alpha_{13}}), \tag{3}$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$. Three mixing angles θ_{ij} vary between 0 and $\pi/2$. The CP violating phases vary between 0 and 2π.

The investigation of neutrino oscillations provides pieces of information about the mixing angles and the differences of masses squared Δm^2_{ij}. It gives us, however, no information about the absolute masses and the possible CP phases, which do not affect the oscillation probabilities. To obtain the absolute values of neutrino masses, one has to look for the neutrinoless double beta decay ($0\nu2\beta$),[3] investigate the end-point of the beta decay spectrum in Tritium,[4] or relay on the rather rough cosmological models of supernova explosions, large scale structure of the Universe etc.[5] The aforementioned $0\nu2\beta$ process is at present the only one known, capable of distinguishing between Majorana and Dirac neutrinos, which makes the searches for $0\nu2\beta$ particularly important.

Another problem, which is not resolved by the oscillation experiments, is the hierarchy of masses. From the differences of masses squared one cannot deduce the actual alignment of m_i. In agreement with the present experimental knowledge are two scenarios:

(i) the so-called normal hierarchy scenario (NH), which is defined by

$$m_1 \ll m_2 \ll m_3,$$

(ii) and the inverted hierarchy (IH), in which m_3 is the lightest one,

$$m_3 \ll m_1 < m_2.$$

At present we have no hint which of the hierarchies is realized in nature, so both of them have to be considered.

The neutrinoless double beta decay is a second order process, forbidden in the standard model due to lepton number violation ($\Delta L = 2$). It may occur only if the neutrinos are Majorana particles and some mechanism of lepton number violation is introduced. These conditions are fullfilled by many extensions of the SM, like the R-parity violating MSSM and others. Therefore the $0\nu2\beta$ process, as the possible source of information about the physics beyond SM, is intensively investigated theoretically and searched for in experiments. The half-life of $0\nu2\beta$ is proportional to the so-called effective neutrino mass, which is just the $\mathcal{M}_{ee}$ element of the neutrino mass matrix Eq. (2). It is a common belief that the confirmation of neutrino oscillations gives a strong back-up for the $0\nu2\beta$ decay. However, in such discussions one usually forgets about the possible CP phases. In the present paper

we show, that the CP phases may in fact completely suppress the $0\nu 2\beta$ process, while still being in agreement with all the oscillation data.

2. Calculations and Results

The best-fit neutrino oscillation parameters may be summarized as follows:[1]

$$\Delta m_{23}^2 = 2.1 \times 10^{-3}\ \mathrm{eV}^2, \qquad \sin^2 2\theta_{23} = 1.00,$$
$$\Delta m_{12}^2 = 7.1 \times 10^{-5}\ \mathrm{eV}^2, \qquad \tan^2 \theta_{12} = 0.40.$$

For the angle θ_{13} only the upper bound is known. From exclusion plot obtained from the data of the reactor experiment CHOOZ[2] we have

$$\sin^2 \theta_{13} \leq 0.05 \qquad (90\%\ \mathrm{c.l.}),$$

with zero being the best-fit value.

The elements of the neutrino mass matrix $\mathcal{M}$ become now a rather complicated functions of the phases. Let us start with the case, in which we take $\sin^2\theta_{13} = 0$. Now, directly from Eqs. (2) and (3) one can obtain the following expressions:

$$\mathcal{M}_{ee} = c_{12}^2 m_1 + s_{12}^2 m_2 e^{-\mathrm{i}2\alpha_{12}} \tag{5}$$
$$\mathcal{M}_{e\mu} = -\mathcal{M}_{e\tau} = \frac{1}{\sqrt{2}} c_{12}s_{12}\left(-m_1 + m_2 e^{-\mathrm{i}2\alpha_{12}}\right), \tag{6}$$
$$\mathcal{M}_{\mu\mu} = \mathcal{M}_{\tau\tau} = \frac{1}{2}\left(s_{12}^2 m_1 + c_{12}^2 m_2 e^{-\mathrm{i}2\alpha_{12}} + m_3 e^{-\mathrm{i}2\alpha_{13}}\right), \tag{7}$$
$$\mathcal{M}_{\mu\tau} = -\frac{1}{2}\left(s_{12}^2 m_1 + c_{12}^2 m_2 e^{-\mathrm{i}2\alpha_{12}} - m_3 e^{-\mathrm{i}2\alpha_{13}}\right). \tag{8}$$

The elements $\mathcal{M}_{\alpha\beta}$ are complex so the physically relevant quantities are $|\mathcal{M}_{\alpha\beta}|$. The strategy now is, that we want to find such combination of the lightest neutrino mass (m_1 in NH and m_3 in IH) and CP phases to obtain $|\mathcal{M}_{\alpha\beta}| = 0$. Because $|\mathcal{M}_{\alpha\beta}| = [\Re(\mathcal{M}_{\alpha\beta})^2 + \Im(\mathcal{M}_{\alpha\beta})^2]^{1/2}$, where $\Re$ and $\Im$ stand for real and imaginary parts, respectively, we are looking for solutions of the set of equations:

$$\begin{cases} \Re(\mathcal{M}_{\alpha\beta}) = 0 \\ \Im(\mathcal{M}_{\alpha\beta}) = 0 \end{cases}$$

for each element in the case of NH and IH.

Our results are presented in Tab. 1. As one sees, in the case of NH only the

Table 1. Results for the case $s_{13} = 0$.

Element	Hierarchy	Mass
$\lvert\mathcal{M}_{ee}\rvert = 0$	NH	$m_1 = 3.557 \times 10^{-3}$ eV
$\lvert\mathcal{M}_{\mu\mu}\rvert = 0$	IH	$m_3 \geq 2.271 \times 10^{-2}$ eV
$\lvert\mathcal{M}_{\tau\tau}\rvert = 0$	IH	$m_3 \geq 2.271 \times 10^{-2}$ eV
$\lvert\mathcal{M}_{\mu\tau}\rvert = 0$	IH	$m_3 \geq 2.272 \times 10^{-2}$ eV

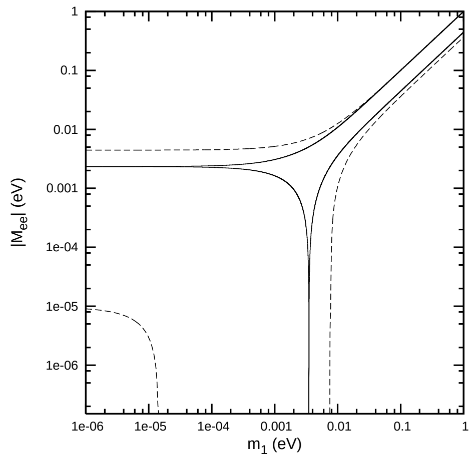

Fig. 1. $|\mathcal{M}_{ee}|$ as the function of lightest neutrino mass m_1 (normal hierarchy scenario). The dashed line corresponds to $\sin\theta_{13} = 0.05$, the solid line to $\sin\theta_{13} = 0$.

element $\mathcal{M}_{ee}$ may be zero, and this situation requires a precisely fine-tuned value of m_1. The corresponding phase α_{12} is given by the condition $\cos(2\alpha_{12}) = -1$. Other elements, of less interest for us, has been calculated as well. It turned out that it is impossible to find zero solutions for the $e\mu$ and $e\tau$ elements, while for the remaining ones there are solutions in the case of IH only. One has to bear in mind, that the lower bounds on m_3 come out from our calculations but do not take into account other constraints on neutrino masses. For example the global astrophysical and cosmological limit on the sum of all three masses $\sum m_i$ is roughly given by 1 eV, which means that m_3 cannot exceed this value. Notice also that, for obvious reasons, it is impossible to have all diagonal elements being zero at one time.

In the case in which $s_{13} \neq 0$, the expressions for $\mathcal{M}_{\alpha\beta}$ become very complicated functions.[6] Some of them depend simultaneously on all three phases which makes their analysis nearly impossible, also numerically. Fortunately, the ee element remains relatively simple:

$$\mathcal{M}_{ee} = c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{-\mathrm{i}2\phi_2} + s_{13}^2 m_3 e^{-\mathrm{i}2(\alpha_{13}-\delta)}. \quad (9)$$

Assuming the maximal allowed value $s_{13}^2 = 0.05$ one obtains the zero solution for NH and

$$8.047 \times 10^{-5}\ \mathrm{eV} \leq m_1 \leq 7.473 \times 10^{-3}\ \mathrm{eV}.$$

One notices that, as expected, this solution contains the one obtained in the simpler case $s_{13} = 0$. The possible values of $|\mathcal{M}_{ee}|$ are depicted on Fig. 1.

3. Conclusions

We have shown that the popular practice of neglecting CP phases in the neutrino mass matrix may have severe consequences. In particular, for certain combination of parameters (phases and the mass of the lightest neutrino), weak processes like the neutrinoless double beta decay may be suppressed. However, the inclusion of CP phases in the calculations is a highly non-linear problem, which may be a big obstacle. It is thus desirable to find a method of determining the possible CP violation in the neutrino sector experimentally.

It is interesting to note, that the obtained values of m_1 which give zero solutions for the $\mathcal{M}_{ee}$ element are within the range of experimental and theoretical bounds (roughly speaking they do not exceed the limit of 1 eV).

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